LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2025**

PST 2602 – MODERN PROBABILITY THEORY

Date: 07-05-2025 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A

Answer ANY FOUR of the following **4 x 10 = 40 Marks**

1. State and prove the continuity property of probability.
2. State and prove the necessary and sufficient condition for n random variables to be independent.
3. Define the 3 types of random variables and state with proof all the properties of a simple random variable.
4. State and prove Basic inequality.
5. Show that convergence in probability implies convergence in distribution.
6. Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$. Then
   (a) $a\,X_n \xrightarrow{P} a\,X$
   (b) $X_n + Y_n \xrightarrow{P} X + Y$
   (c) $X_n\ Y_n \xrightarrow{P} X\ Y$
   (d) $\dfrac{X_n}{Y_n} \xrightarrow{P} \dfrac{X}{Y}$
7. State and prove Kolmogorov's strong law of large numbers.
8. State and prove the conditions under which WLLN holds.

SECTION B

Answer ANY THREE of the following **3 x 20 = 60 Marks**

9. State and prove the necessary and sufficient condition for a function F to be the distribution function of a random variable.
10. Let F be the distribution function on R given by,

$$F(x)=\begin{cases} o,\ if\ \ x<-1 \\ 1+x,\ if\ \ -1\le x<0 \\ 2+x^2,\ if\ \ 0\le x<2 \\ 9,\quad if\ x\ge 2 \end{cases}$$

If μ is a Lebesgue-Stieltjes measure corresponding to F, compute the measure of each of the following sets. (i) {2}, (ii) [-1/2, 3), (iii) (-1, 0]U(1, 2), (iv) [0,1/2)U(1,2).

11. State the inversion theorem for discrete and continuous case and find
    (a) Characteristic function φ (u) of normal distribution
    (b) The distribution if $\varphi(u)=e^{-|t|}$, $-\infty<t<\infty$
12. State and prove weak law of large numbers for the iid case.
13. State and prove the Lindeberg-Levi central limit theorem clearly explaining the assumptions.
14. (i) State and prove Markov's theorem.

    (ii) Define the characteristic function of a random variable and show that it is continuous.

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